Math 53, Discussions 116 and 118 **2022-10-26**

More triple integrals, and cylindrical

Answers

Questions

Question 1. Write (but do not evaluate) a triple integral for the volume of the region bounded by the planes $y = 0, z = 0, x + y = 2$ and the cylinder $y^2 + z^2 = 1$ in the first octant.

Question 2. Express (but do not evaluate) the following triple integral in cylindrical coordinates.

$$\int_{-1}^{1} \int_{0}^{\sqrt{1-y^2}} \int_{0}^{9-x^2-y^2} \sqrt{x^2 + y^2} \, \mathrm{d}z \, \mathrm{d}x \, \mathrm{d}y.$$

Below are brief answers to the worksheet exercises. If you would like a more detailed solution, feel free to ask me in person. (Do let me know if you catch any mistakes!)

Answers to questions

Question 1. There are a lot of correct answers for this problem! Here are some that we looked at in class.

We could use cylindrical coordinates, but with x playing the role of z, i.e. $x = x$, $y = r\cos\theta, z = r\sin\theta$. This would give

$$\int_0^{\pi/2}\int_0^1\int_0^{2-r\cos\theta} r\,\mathrm{d}x\,\mathrm{d}r\,\mathrm{d}\theta.$$

Another option, using the dx order first:

$$\int_0^1\int_0^{2-y}\int_0^{\sqrt{1-y^2}} \mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y.$$

It is not convenient to integrate dy first, because this would require splitting up the region.

Question 2. I drew a picture of the region in class.

$$\int_{-\pi/2}^{\pi/2}\int_0^1\int_0^{9-r^2} r^2\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta.$$